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Instantaneous optimal method for vibration control of linear sampled-data systems with time delay in control

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Abstract

In an actively controlled system, time delay exists inevitably. Neglecting time delay may cause degradation of control performance or even induce instability to the dynamic system. In this paper, instantaneous optimal control method for vibration control of linear sampled-data systems with time delay in control is investigated. By a peculiar integral transformation, the first order state equation with time delay is transformed into the standard first order state equation, which contains no time delay. Then the optimal controller is designed based on the numerical algorithm of the regular fourth order Runge–Kutta method. Since the obtained controller contains integral term, which is not practical for control implementation, the numerical algorithm for this integral term is investigated too. Since the controller is deduced directly from the time-delay differential equation, the control method presented is prone to guarantee system stability. Thus the presented control method can be applicable to the case of large time delay. The performance of the control method is demonstrated by numerical simulation. Simulation results indicate that this control method is feasible and is an attractive strategy for dealing with the time delay in vibration control systems and is effective in suppressing maximum structural responses. Instability in structural responses may occur if the systems with time delay are controlled using the controller designed in the case of no time delay.

1. Introduction

In recent years, the technique of active vibration control has been developing rapidly, and many active control methods have been used in practical engineering. Results both in laboratory demonstrations and in actual measurements for practical engineering demonstrate that vibration suppression by active control is emerging as a powerful technique to improve the performance of

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structures against earthquakes, wind and other dynamic excitations. Meanwhile, many problems that affect this technique towards large-scale practical application have been found [1,2]. Time delay is one of these problems which needs a serious attention.

Time delay exists inevitably in active control systems. Time delay occurs mainly because of: (1) time taken in on-line data acquisition from sensors at different locations of the systems, (2) time taken in filtering, processing of these data for required control force calculation and transmission of the control force to the actuator, (3) time taken by the actuator in building up the required control force. Because of the time delay, unsynchronized control force may be applied to the systems, which may cause degradation in the efficiency of control or may render the control systems unstable [1,2].

Time-delay problem has been studied by researchers in many fields of applications, such as aeronautical, astronautical, mechanical, chemical and electrical engineering. A great deal of literature has been available in different disciplines [3–6] and various methodologies to deal with the problem of a system time delay have been available in the literature. For the particular application of vibration control, the technique of time delay compensation [7–10] has been investigated for reducing or eliminating the effect of the time delay. In the time-delay compensation, the controller is designed without considering the existence of the time delay and then the control feedback gain is modified with considering the time delay. However, system stability is not guaranteed as the time delay becomes larger [11]. Therefore, this technique is in general available for the case of small time delay. Except the technique of time-delay compensation, another method, in which controller is deduced directly from time-delay differential equation, can be available for dealing with the time delay in vibration system. In Ref. [11], the discrete optimal control method is studied for an actively controlled system with time delay in control. Since the time delay is incorporated in the mathematical model for the structural control system throughout the derivation of the proposed method, system performance and dynamic stability are guaranteed. Thus this method is suitable not only for small time delay but also for large time delay. This kind of design method which considers time delay at the very beginning of the derivation of control algorithm is more feasible and is an attractive approach in control design. On the other hand, in addition to the classical optimal control method, many other control methods, including instantaneous optimal control [12–14], variable structure control [15], modal control [16], H_∞ control [17], and so on have been investigated before. To guarantee control performance and system stability, it is essential to consider the time delay problem in control design when using these control methods because of the inevitable existence of the time delay and its concern of system dynamic stability.

The instantaneous optimal control method is proposed by Yang and is investigated for linear structures [12,13]. The numerical results indicate that this method is effective in suppressing maximum responses of the structures and is feasible for practical application. The purpose of this paper is to investigate an instantaneous optimal control method for vibration control of linear sampled-data systems with time delay in control, in which the optimal controller is designed with considering the time delay in the time-delay differential equation. By the peculiar integral transformation given by Kwon [18], the differential equation with time delay can be changed into that which contains no time delay apparently. Then the controller may be designed according to the instantaneous optimal control method given by Yang [12–14]. Substituting the integral transformation into the obtained controller, an integral term may appear in the controller, which is not practical for control implementation. The numerical algorithm for this integral term is investigated in this paper too. Finally, numerical simulations are carried out for two structural models to demonstrate the performance and feasibility of the presented control method.

This paper is organized as follows. Section 2 first briefly presents the system motion equation with an explicit time delay in the differential equation, and then introduces a peculiar integral transformation that reformulates the system dynamics into a standard first order differential equation without any time delay. The numerical algorithm for this standard first order differential equation using the regular fourth order Runge–Kutta method is presented subsequently. The proposed control design is presented in Section 3, including the controller design and a numerical algorithm of its implementation. Section 4 provides the simulation and comparison studies of two structural models using the proposed control method. Finally, a conclusion remark is given in Section 5.

2. Motion equation

Consider a linear structure modelled by an n-degree-of-freedom lumped mass–spring–dashpot system with time delay in control. The matrix equation of motion of the structural system is written as

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X} = \mathbf{H}_1\mathbf{p}(t) + \mathbf{H}_2\mathbf{U}(t-\lambda), \tag{1}$$

where $\mathbf{X} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$ is an n-dimensional vector of displacement; $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are $(n \times n)$ mass, damping and stiffness matrices, respectively; $\mathbf{p}(t)$ is an m-dimensional vector of external excitations, $\mathbf{H}_1$ is a $(n \times m)$ matrix denoting the location of the external excitations; $\mathbf{U}(t-\lambda)$ is an r-dimensional vector of controllers, in which λ is the time delay; $\mathbf{H}_2$ is a $(n \times r)$ matrix denoting the location of the controllers.

In the state-space representation, Eq. (1) becomes

$$\dot{\mathbf{Z}}(t) = \mathbf{A}\mathbf{Z}(t) + \mathbf{B}\mathbf{U}(t-\lambda) + \mathbf{P}(t), \tag{2}$$

where $\mathbf{Z}(t)$ is a $2n$-dimensional state vector; $\mathbf{A}$ is a $(2n \times 2n)$ system matrix; $\mathbf{B}$ is a $(2n \times r)$ matrix and $\mathbf{P}(t)$ is a $2n$-dimensional excitation vector, respectively, given by

$$\mathbf{Z}(t) = \begin{bmatrix} \mathbf{X}(t) \\ \dot{\mathbf{X}}(t) \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_n \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \quad \mathbf{P}(t) = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{H}_1\mathbf{p}(t) \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{H}_2 \end{bmatrix},$$

where $\mathbf{I}_n$ is a $(n \times n)$ identity matrix.

Making the transformation [18]

$$\mathbf{Y}(t) = \mathbf{Z}(t) + \int_{-\lambda}^{0} \mathrm{e}^{-A(\eta+\lambda)}\mathbf{B}\mathbf{U}(t+\eta)\,\mathrm{d}\eta, \tag{3}$$

Eq. (2) can be written as

$$\dot{\mathbf{Y}}(t) = \mathbf{A}\mathbf{Y}(t) + \mathbf{B}(\mathbf{A})\mathbf{U}(t) + \mathbf{P}(t) \tag{4}$$

in which

$$\mathbf{B}(\mathbf{A}) = \mathrm{e}^{-A\lambda}\mathbf{B}, \tag{5}$$

where $\mathbf{B}(\mathbf{A})$ is a $(2n \times r)$ matrix. From Ref. [19], we can obtain that Eq. (4) is controllable if Eq. (2) is absolutely controllable.

When the fourth order Runge–Kutta method is used to solve Eq. (4), we have the following algorithm [12–14]:

$$\mathbf{Y}(t) = \mathbf{Y}(t - 2\Delta t) + \tfrac{1}{6}(\mathbf{A}_0 + 2\mathbf{A}_1 + 2\mathbf{A}_2 + \mathbf{A}_3), \tag{6}$$

where Δt is the integration time step; and $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$ and $\mathbf{A}_3$ are all $2n$-dimensional vectors and can be written as follows:

$$\mathbf{A}_0 = 2\Delta t[\mathbf{A}\mathbf{Y}(t - 2\Delta t) + \mathbf{B}(\mathbf{A})\mathbf{U}(t - 2\Delta t) + \mathbf{P}(t - 2\Delta t)], \tag{7a}$$

$$\mathbf{A}_1 = 2\Delta t[\mathbf{A}\mathbf{Y}(t - 2\Delta t) + 0.5\mathbf{A}\mathbf{A}_0 + \mathbf{B}(\mathbf{A})\mathbf{U}(t - \Delta t) + \mathbf{P}(t - \Delta t)], \tag{7b}$$

$$\mathbf{A}_2 = 2\Delta t[\mathbf{A}\mathbf{Y}(t - 2\Delta t) + 0.5\mathbf{A}\mathbf{A}_1 + \mathbf{B}(\mathbf{A})\mathbf{U}(t - \Delta t) + \mathbf{P}(t - \Delta t)], \tag{7c}$$

$$\mathbf{A}_3 = 2\Delta t[\mathbf{A}\mathbf{Y}(t - 2\Delta t) + \mathbf{A}\mathbf{A}_2 + \mathbf{B}(\mathbf{A})\mathbf{U}(t) + \mathbf{P}(t)]. \tag{7d}$$

Substituting Eqs. (7a)–(7d) into Eq. (6), we have

$$\mathbf{Y}(t) = \mathbf{D}(t - 2\Delta t, t - \Delta t) + \frac{\Delta t}{3}[\mathbf{B}(\mathbf{A})\mathbf{U}(t) + \mathbf{P}(t)], \tag{8}$$

where $\mathbf{D}(t - 2\Delta t, t - \Delta t)$ is a $2n$-dimensional vector and can be written as

$$\begin{aligned}\mathbf{D}(t - 2\Delta t, t - \Delta t) = {} & (\mathbf{I}_{2n} + 2\Delta t\mathbf{A})\mathbf{Y}(t - 2\Delta t) + \tfrac{1}{3}\Delta t\{\mathbf{A}(\mathbf{A}_0 + \mathbf{A}_1 + \mathbf{A}_2) \\ & + \mathbf{B}(\mathbf{A})[\mathbf{U}(t - 2\Delta t) + 4\mathbf{U}(t - \Delta t)] + [\mathbf{P}(t - 2\Delta t) + 4\mathbf{P}(t - \Delta t)]\},\end{aligned} \tag{9}$$

where $\mathbf{I}_{2n}$ is a $(2n \times 2n)$ identity matrix.

3. Instantaneous optimal control

The instantaneous optimal control method was proposed by Yang [12,13] for linear structures and was extended later to non-linear and hysteretic structures [14]. In this section, we study the control design and control implementation when using the instantaneous optimal control method for the above time-delay dynamic system.

3.1. Design of controller

The time-dependent quadratic objective function is given by

$$J(t) = \mathbf{Y}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{Y}(t) + \mathbf{U}^{\mathrm{T}}(t)\mathbf{R}\mathbf{U}(t), \tag{10}$$

where $\mathbf{Q}$ is a $(2n \times 2n)$ positive-semidefinite weighting matrix; and $\mathbf{R}$ is a $(r \times r)$ positive-definite weighting matrix, respectively, representing the relative importance of the response vector $\mathbf{Y}(t)$ and the control vector $\mathbf{U}(t)$. The two weighting matrices are often determined by trial runs until the selected values result in significant reduction of responses while control forces are also acceptable. In general, large values of $\mathbf{Q}$ result in smaller values of the responses of the structure.

The implication of minimizing the objective function given by Eq. (10) is that the performance index $J(t)$ is minimized at every time instant t.

To minimize the objective function $J(t)$, the Hamilton function H is obtained by introducing a $2n$-dimensional Lagrangian multiplier vector $\mathbf{L}(t)$:

$$H = \mathbf{Y}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{Y}(t) + \mathbf{U}^{\mathrm{T}}(t)\mathbf{R}\mathbf{U}(t) + \mathbf{L}^{\mathrm{T}}(t)\left\{\mathbf{Y}(t) - \mathbf{D}(t-2\Delta t, t-\Delta t) - \frac{\Delta t}{3}[\mathbf{B}(\mathbf{A})\mathbf{U}(t) + \mathbf{P}(t)]\right\}. \tag{11}$$

The necessary conditions for minimizing $J(t)$, subjected to the constraint of Eq. (8), can be written as

$$\frac{\partial H}{\partial \mathbf{Y}(t)} = \mathbf{0}, \quad \frac{\partial H}{\partial \mathbf{U}(t)} = \mathbf{0}, \quad \frac{\partial H}{\partial \mathbf{L}(t)} = \mathbf{0}. \tag{12}$$

Substituting Eq. (11) into the first two expressions of Eq. (12), we have

$$2\mathbf{Q}\mathbf{Y}(t) + \mathbf{L}(t) = \mathbf{0}, \tag{13}$$

$$2\mathbf{R}\mathbf{U}(t) - \frac{\Delta t}{3}[\mathbf{B}(\mathbf{A})]^{\mathrm{T}}\mathbf{L}(t) = \mathbf{0}. \tag{14}$$

Substituting Eq. (11) into the third expression of Eq. (12) deduces Eq. (8). From Eqs. (13) and (14), the optimal controller can be obtained as

$$\mathbf{U}(t) = -\frac{\Delta t}{3}\mathbf{R}^{-1}[\mathbf{B}(\mathbf{A})]^{\mathrm{T}}\mathbf{Q}\mathbf{Y}(t). \tag{15}$$

Substituting Eq. (15) into Eq. (8), we have

$$\mathbf{Y}(t) = \left[\mathbf{I}_{2n} + \left(\frac{\Delta t}{3}\right)^2[\mathbf{B}(\mathbf{A})]\mathbf{R}^{-1}[\mathbf{B}(\mathbf{A})]^{\mathrm{T}}\mathbf{Q}\right]^{-1}\left[\mathbf{D}(t-2\Delta t, t-\Delta t) + \frac{\Delta t}{3}\mathbf{P}(t)\right]. \tag{16}$$

Substituting Eq. (16) into Eq. (15), we have

$$\mathbf{U}(t) = -\frac{\Delta t}{3}\mathbf{R}^{-1}[\mathbf{B}(\mathbf{A})]^{\mathrm{T}}\mathbf{Q}\left[\mathbf{I}_{2n} + \left(\frac{\Delta t}{3}\right)^2[\mathbf{B}(\mathbf{A})]\mathbf{R}^{-1}[\mathbf{B}(\mathbf{A})]^{\mathrm{T}}\mathbf{Q}\right]^{-1} \times \left[\mathbf{D}(t-2\Delta t, t-\Delta t) + \frac{\Delta t}{3}\mathbf{P}(t)\right]. \tag{17}$$

It should be noted that for the regular fourth order Runge–Kutta method, the integration time step is $\Delta\tau$, which is twice Δt used in Eqs. (6)–(9), i.e., $\Delta\tau = 2\Delta t$. In other words, the computation step in the current numerical scheme is Δt, which is one-half that used in the regular fourth order Runge–Kutta method.

When there is no time delay in the system, namely $\lambda = 0$, from Eqs. (3) and (5), we have $\mathbf{B}(\mathbf{A}) = \mathbf{B}$ and $\mathbf{Y}(t) = \mathbf{Z}(t)$ [here the integral term in Eq. (3) is equal to zero], so the controller can be written as $\mathbf{U}(t) = -(\Delta t/3)\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{Q}\mathbf{Z}(t)$. In this case, we can observe from Eqs. (9) and (17) that the controller $\mathbf{U}(t)$ at time instant t is dependent on the external excitation $\mathbf{p}(t)$ at time instant t, the controller $\mathbf{U}(t-\Delta t)$ and the external excitation $\mathbf{p}(t-\Delta t)$ at time instant $(t-\Delta t)$, and the state vector $\mathbf{Z}(t-2\Delta t)$, the controller $\mathbf{U}(t-2\Delta t)$ and the external excitation $\mathbf{p}(t-2\Delta t)$ at time instant

$(t-2\Delta t)$. When $\lambda \neq 0$, $\mathbf{U}(t)$ is relative to those mentioned above and the integral term in Eq. (3) as well. The computation of this term will be discussed in the following.

3.2. Control implementation

When the instantaneous optimal control method is used with considering the existence of the time delay, implementation of the controller given by Eq. (17) depends on the computation of the integral term in Eq. (3). Numerical algorithm for this integral term is investigated in this section.

Let

$$\mathbf{Z}_0(t) = \int_{-\lambda}^{0} \mathrm{e}^{-A(\eta+\lambda)}\mathbf{B}\mathbf{U}(t+\eta)\,\mathrm{d}\eta. \tag{18}$$

Assume that the data sampling period $\bar{T}$ is identical with the computation step, i.e., $\bar{T} = \Delta t$. Also, assume the time delay λ can be written as

$$\lambda = l\bar{T} - \bar{m}, \tag{19}$$

where $l > 0$ is a integer number; and $0 \leqslant \bar{m} < \bar{T}$. When $\bar{m} = 0$, the time delay is integer times of the sampling period. When $\bar{m} \neq 0$, the time delay is non-integer times of the sampling period.

Zero order holder is used in the structure, i.e.,

$$\mathbf{U}(t) = \mathbf{U}(k\bar{T}), \quad k\bar{T} \leqslant t < (k+1)\bar{T}. \tag{20}$$

Eq. (20) represents that the actuators in the structure exert constant control forces on the structure during two adjoining sampling points.

Noting that numerical computation is carried out only on every sampling point, Eq. (18) can be written as

$$\begin{aligned}
\mathbf{Z}_0(t) &= \int_{-(l\bar{T}-\bar{m})}^{0} \mathrm{e}^{-\mathbf{A}(l\bar{T}-\bar{m})}\mathrm{e}^{-\mathbf{A}\eta}\mathbf{B}\mathbf{U}(t+\eta)\,\mathrm{d}\eta \\
&= \mathrm{e}^{-\mathbf{A}(l\bar{T}-\bar{m})}\left[\int_{-(l\bar{T}-\bar{m})}^{-(l-1)\bar{T}} \mathrm{e}^{-\mathbf{A}\eta}\mathbf{B}\mathbf{U}(t+\eta)\,\mathrm{d}\eta + \int_{-(l-1)\bar{T}}^{-(l-2)\bar{T}} \mathrm{e}^{-\mathbf{A}\eta}\mathbf{B}\mathbf{U}(t+\eta)\,\mathrm{d}\eta \right. \\
&\quad \left. + \cdots + \int_{-\bar{T}}^{0} \mathrm{e}^{-\mathbf{A}\eta}\mathbf{B}\mathbf{U}(t+\eta)\,\mathrm{d}\eta\right] \\
&= \mathrm{e}^{-\mathbf{A}(l\bar{T}-\bar{m})}\left[\mathrm{e}^{\mathbf{A}(l\bar{T}-\bar{m})}\cdot\int_{0}^{\bar{T}-\bar{m}} \mathrm{e}^{-\mathbf{A}\eta_1}\,\mathrm{d}\eta_1\,\mathbf{B}\mathbf{U}(t-l\bar{T}) + \mathrm{e}^{\mathbf{A}(l-1)\bar{T}}\int_{0}^{\bar{T}} \mathrm{e}^{-\mathbf{A}\eta_2}\,\mathrm{d}\eta_2\,\mathbf{B}\mathbf{U}[t-(l-1)\bar{T}] \right. \\
&\quad \left. + \cdots + \mathrm{e}^{\mathbf{A}\bar{T}}\int_{0}^{\bar{T}} \mathrm{e}^{-\mathbf{A}\eta_l}\,\mathrm{d}\eta_l\,\mathbf{B}\mathbf{U}(t-\bar{T})\right].
\end{aligned} \tag{21}$$

Let

$$\mathbf{F}(\xi) = \mathrm{e}^{\mathbf{A}\xi}, \tag{22}$$

$$\mathbf{G}(\xi) = \int_{0}^{\xi} \mathrm{e}^{-\mathbf{A}\theta}\,\mathrm{d}\theta. \tag{23}$$

In consideration of $\bar{T} = \Delta t$, Eq. (21) can be written as

$$\begin{aligned}\mathbf{Z}_0(t) = {}& \mathbf{I}_{2n\times 2n}\mathbf{G}(\Delta t - \bar{m})\mathbf{U}(t - l\Delta t) + \mathbf{F}(\bar{m} - \Delta t)\mathbf{G}(\Delta t)\mathbf{U}[t - (l-1)\Delta t] \\ & + \mathbf{F}(\bar{m} - 2\Delta t)\mathbf{G}(\Delta t)\mathbf{U}[t - (l-2)\Delta t] + \cdots + \mathbf{F}[\bar{m} - (l-1)\Delta t]\mathbf{G}(\Delta t)\mathbf{U}(t - \Delta t). \end{aligned} \tag{24}$$

When $\bar{m} = 0$, Eq. (24) can be written as

$$\begin{aligned}\mathbf{Z}_0(t) = {}& \mathbf{I}_{2n\times 2n}\mathbf{G}(\Delta t)\mathbf{U}(t - l\Delta t) + \mathbf{F}(-\Delta t)\mathbf{G}(\Delta t)\mathbf{U}[t - (l-1)\Delta t] \\ & + \mathbf{F}(-2\Delta t)\mathbf{G}(\Delta t)\mathbf{U}[t - (l-2)\Delta t] + \cdots + \mathbf{F}[-(l-1)\Delta t]\mathbf{G}(\Delta t)\mathbf{U}(t - \Delta t). \end{aligned} \tag{25}$$

From Eqs. (24) and (25), we can observe that, in every step of numerical computation for the controller given by Eq. (17), it contains the linear combination of former l steps of control.

$\mathbf{G}(\xi)$ given by Eq. (23) can be determined according to the following formula [20]:

$$\mathbf{G}(\xi) = \int_0^{\xi} \mathrm{e}^{-\mathbf{A}\theta}\,\mathrm{d}\theta = \sum_{n=1}^{\infty} \frac{(-\mathbf{A})^{n-1}\xi^n}{n!}. \tag{26}$$

When ξ is given, $\mathbf{G}(\xi)$ will tend to a constant matrix after limited steps of iterative computation.

4. Numerical examples

To illustrate the application of the presented control method and its performance, simulation results for a three-story and a six-story seismic-excited structural models are presented in this section. The model sketch is shown by Fig. 1. The El Centro earthquake and the Tianjin earthquake (in China) are used as the input excitations, respectively.

4.1. Example 1: three-story structural model subjected to the El Centro earthquake excitation

A three-story model studied by Yang et al. [15] is considered as shown in Fig. 1, in which every story unit is identically constructed. The mass, stiffness and damping coefficient of each story unit are $m_i = 1000\,\mathrm{kg}$, $k_i = 980\,\mathrm{kN/m}$, and $c_i = 1.407\,\mathrm{kN\,s/m}$, respectively ($i = 1-3$). The sampling period and numerical computation step are both taken by 0.01 s, i.e., $\bar{T} = \Delta t = 0.01$ s. An active control force is applied on the first-story unit, as shown in Fig. 1.

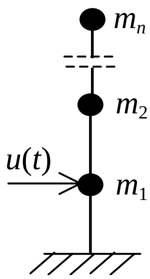

Fig. 1. Structural model.

As such in Ref. [21], the variable x_i ($i = 1$–3) in Eq. (1) is designated to be the interstory drift of the ith story unit of the structure, representing the relative displacement of the ith story unit with respect to the $(i-1)$th story unit. The variables $\dot{x}_i$ and $\ddot{x}_i$ are the corresponding velocity and acceleration variables. The matrices **M**, **C**, **K**, $\mathbf{H}_1$ and $\mathbf{H}_2$ in Eq. (1) are

$$\mathbf{M} = 1000 \times \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad \mathbf{C} = 1407 \times \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{K} = 980{,}000 \times \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{H}_1 = -1000 \times \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{H}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

respectively.

The El Centro earthquake scaled to a maximum acceleration of $0.12g$ is used as the input excitation herein. The earthquake episode is 8 s. Time history of the earthquake is shown in Fig. 2(a).

The maximum interstory drifts x_i and the maximum absolute floor accelerations $\ddot{x}_{ai}$ of every story unit, without control for the structure, are shown in columns (2) and (3) of Table 1.

For active structural control of the building structure, the hydraulic type of actuator is popularly used as a control-force device because of the large weight of the structure. However, the time-delay problem exists obviously in this type of actuator. Here consider the case of $\lambda = 0.1$ s. Namely, the delayed time in the control system is 0.1 s. The dimension of the weighting matrix **Q** given in Eq. (10) is (6×6). Because there is only one actuator in the structure, the weighting matrix **R** in Eq. (10) consists of one element and is a scalar. All elements of the weighting matrix **Q** are set to be zero except that elements of the fourth row of **Q** are chosen to be $\mathbf{Q}_{4j} = [10^3, 10^3, 10, 10, 1, 1]$. The scalar R is assigned to be $R = 1 \times 10^{-9}$. When the proposed

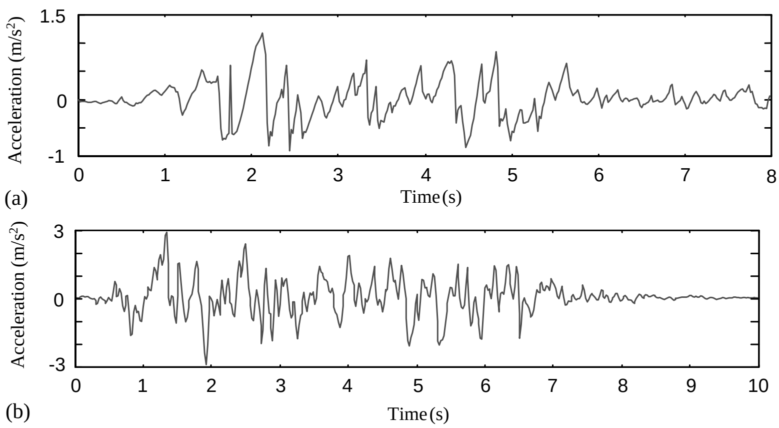

Fig. 2. Time histories of the El Centro earthquake and the Tianjin earthquake. (a) The El Centro earthquake, (b) The Tianjin earthquake.

Table 1
Maximum response quantities and maximum required control force of the three-story model under the El Centro earthquake excitation (x_i: cm, $\ddot{x}_{ai}$: cm/s^2)

Story	No control		Case I ($\lambda = 0.1$ s); $U_{max} = 2452$ N		Case II ($\lambda = 0.3$ s); $U_{max} = 1987$ N		Case III ($\lambda = 0$); $U_{max} = 2452$ N		Case IV ($\lambda = 0$); $U_{max} = 1987$ N	
	x_i	$\ddot{x}_{ai}$	x_i	$\ddot{x}_{ai}$	x_i	$\ddot{x}_{ai}$	x_i	$\ddot{x}_{ai}$	x_i	$\ddot{x}_{ai}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	1.37	330	0.70	235	0.88	232	0.45	190	0.59	203
2	1.04	496	0.41	210	0.58	295	0.53	277	0.59	331
3	0.61	600	0.25	246	0.33	326	0.32	322	0.35	342

control method is used for the structure, the maximum response quantities of every story unit and the maximum required control force U_{max} are shown in columns (4) and (5) of Table 1, denoted by "Case I". Comparing the results of columns (4) and (5) with columns (2) and (3) shows the effectiveness of the proposed control method. Next consider the case of $\lambda = 0.3$ s. Elements of the **Q** matrix are chosen to be $\mathbf{Q}_{4j} = [10^5, 10^4, 10^3, 0, 0, 0]$ with other elements being zero. The scalar R is given by $R = 1.5 \times 10^{-7}$. The simulation results are listed in columns (6) and (7) of Table 1, denoted by "Case II". Compared with the result listed in columns (2) and (3) of Table 1, the results in columns (6) and (7) are also acceptable.

In Refs. [12–14], the instantaneous optimal control method was proposed by Yang and was investigated for application to seismically excited building structures without existence of time delay. If no time delay exists, the controller given by Eq. (15) is the same as that given by Yang [14]. In this case, $\mathbf{B}(\mathbf{A}) = \mathbf{B}$ in Eq. (15). Using this controller with no time delay to control the system with the delayed times $\lambda = 0.1$ and 0.3 s in control, respectively, both the control systems become unstable. Therefore, time-delay problem is of importance for control design since it exists inevitably in an actively controlled system. It is better not to put any control action into the structural system before time delay is properly analyzed and tackled.

Now consider the comparisons of control effectiveness under the same control cost. When no time delay exists, using the controller with $\lambda = 0$ for the system, the maximum response quantities of every story unit and the maximum control forces are listed in columns (8)–(11) of Table 1, denoted by "Case III" and "Case IV". In the calculations for columns (8)–(11), the weighting matrix **Q** is chosen to be the same as Case I and Case II of Table 1, whereas the scalar R is adjusted such that the maximum control forces are $U_{max} = 2452$ and $U_{max} = 1987$ N, respectively. We can observe from the comparisons of columns (4) and (5) with (8) and (9), and columns (6) and (7) with (10) and (11) that, under the same control cost, the control performance of the proposed method is close to that when no time delay exists except that the value of interstory drift of the first story unit is a little bigger. However, the results shown in columns (4) and (5) and columns (8) and (9) of Table 1 are much small compared with that shown in columns (2) and (3).

In addition, extensive numerical simulation indicates that, when the delayed times $\lambda = 0.1$ and 0.3 s exist, respectively, in the control system, instability in structural responses both occurs when the method of time delay compensation [10] is used for the system.

Fig. 3 shows the time histories of the third story unit and the control force when the proposed control method is used for the structure with $\lambda = 0.1\,\text{s}$ in control (Case I of Table 1). Time histories of the third story unit when no time delay exists (Case III of Table 1), and time histories without control for the structure are shown in Fig. 3 for comparison too. We can observe from Fig. 3 that good control effectiveness can be achieved when the proposed control method is used.

4.2. Example 2: six-story structural model subjected to the Tianjin earthquake excitation

The structural model studied by Fayaz et al. [22] is considered herein. This model is a six-story base-isolated structure, in which the first-story unit is the base-isolated system. The mass, stiffness and damping coefficient of each story unit are as follows:

$$m_1 = 6800\,\text{kg},\quad m_i = 5897\,\text{kg}\,(i = 2{-}6);\quad k_1 = 1200\,\text{kN/m},\quad k_2 = 33,732\,\text{kN/m},$$
$$k_3 = 29,093\,\text{kN/m},\quad k_4 = 28,621\,\text{kN/m},\quad k_5 = 24,954\,\text{kN/m},\quad k_6 = 19,054\,\text{kN/m};$$
$$c_1 = 2.4\ \text{kN s/m},\quad c_i = 0.002k_i\,\text{kN s/m}\,(i = 2{-}6).$$

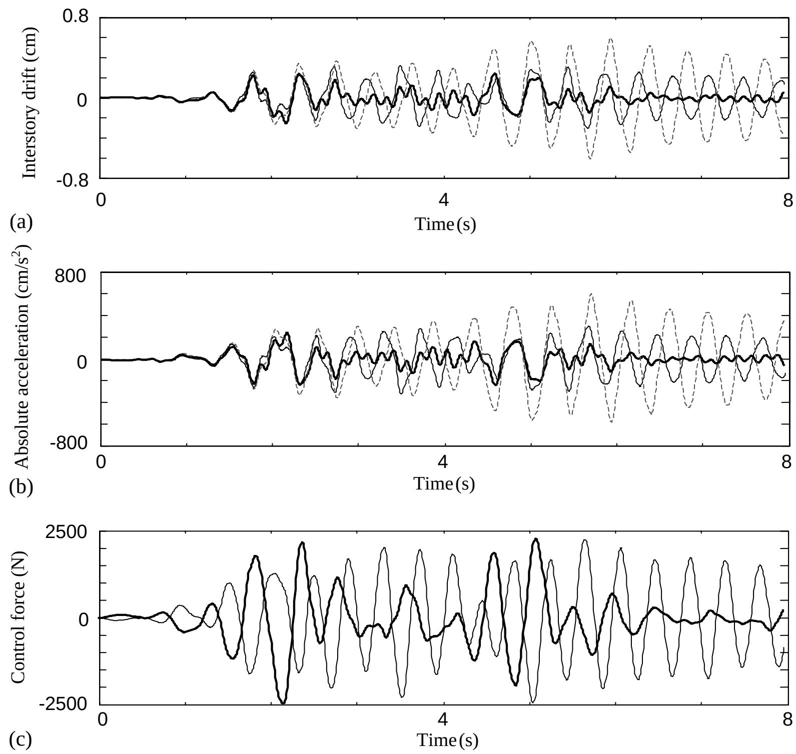

Fig. 3. Time histories of the third story unit and control force under the following three control strategies: no control, instantaneous optimal control without time delay and instantaneous optimal control with time delay: (a) interstory drifts: no control ---------; optimal control without time delay ——; optimal control with time delay ——; (b) absolute acceleration: no control ---------; optimal control without time delay ——; optimal control with time delay ——; (c) control force: optimal control without time delay ——; optimal control with time delay ——.

Table 2
Maximum response quantities and maximum required control force of the six-story model under the Tianjin earthquake excitation (x_i: cm, $\ddot{x}_{ai}$: cm/s^2)

Story	No control		Case I ($\lambda = 0$); $U_{max} = 37.30$ kN		Case II ($\lambda = 0$); $U_{max} = 35.99$ kN		Case III ($\lambda = 0.3$ s); $U_{max} = 45.26$ kN		Case IV ($\lambda = 0.3$ s); $U_{max} = 35.99$ kN	
	x_i	$\ddot{x}_{ai}$	x_i	$\ddot{x}_{ai}$	x_i	$\ddot{x}_{ai}$	x_i	$\ddot{x}_{ai}$	x_i	$\ddot{x}_{ai}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	16.86	523	4.20	207	4.32	208	6.92	213	9.63	266
2	0.49	538	0.17	180	0.17	193	0.16	211	0.22	265
3	0.47	555	0.17	182	0.16	187	0.15	194	0.21	262
4	0.36	569	0.13	206	0.13	204	0.12	187	0.16	254
5	0.28	582	0.11	221	0.11	219	0.11	195	0.13	263
6	0.18	591	0.08	250	0.08	249	0.08	259	0.09	277

The matrices **M**, **C**, **K**, $\mathbf{H}_1$ and $\mathbf{H}_2$ in Eq. (1) have the same constitution with those in Example 1, and omitted herein. Again assume that only the first story unit of the structure is applied with active control force. $\bar{T} = \Delta t = 0.01$ s is used again. The Tianjin earthquake (in China) scaled to a maximum acceleration of $0.3\,g$ is used as the input excitation. The earthquake episode is 10 s. Time history of the Tianjin earthquake is shown in Fig. 2(b).

For the structure equipped with the base-isolated system and with the earthquake ground acceleration given in Fig. 2(b), the maximum interstory drifts and the maximum absolute floor accelerations of every story unit, without control for the structure, are shown in columns (2) and (3) of Table 2. As observed from these two columns, the maximum response quantities of the upper story units upon the base floor are very small. The advantage of using a base isolation system to protect the structure is clearly demonstrated. However, the deformation of the base isolation system shown in the first row of column (2) of Table 2 may be excessive.

To protect the base isolation system, an actuator is connected directly to the base isolation system. Consider the case without existence of time delay in the control system. In this case, $\mathbf{B(A)} = \mathbf{B}$. The dimension of the **Q** matrix in Eq. (10) is (12×12), whereas the weighting matrix **R** consists of only one element and is a scalar. Since there is only one actuator installed on the base floor, only elements in the seventh row and the eighth row of the **Q** matrix are relevant to the controller [see Eq. (15)]. As a result, all elements of the **Q** matrix that have no effect on the controller are set to be zero for convenience. Furthermore, elements in the eighth row of the **Q** matrix will assigned to be zero for simplicity. The values of the seventh row of the **Q** matrix are assigned to be $\mathbf{Q}_{7j} = [0.25, 5.5, 4.5, 3.4, 2.4, 1.4, 3.5, 0.5, 0.5, 0.5, 0.5, 0.4]$. The scalar R is chosen to be 1.3×10^{-11}. Using the optimal controller with no time delay, the maximum response quantities and the maximum required control force are listed in columns (4) and (5) of Table 2, denoted by "Case I". We can observe that a significant reduction of the structure response, in particular the response of the base isolation system, can be achieved using the actuator for the structure. It is further observed from columns (2)–(5) of Table 2 that, for the structure equipped with a base isolation system, the interstory deformation of each story unit of the upper structure is very small compared to that of the base isolation system. Thus, the upper structure upon the base isolation tends to behave like a rigid body. As a result, sensors are not needed for the upper structure. In other words, displacement and velocity sensors are installed on the base isolation system only, i.e., no sensor is installed on the upper structure. To illustrate the control performance in this case, all the elements of the **Q** matrix are set to be zero except $Q(7,1)$ and $Q(7,7)$. For simplicity, the following values are assigned: $Q(7,1) = 0.9$; and $Q(7,7) = 3.4$. The scalar R is the same as the case of full sensors being installed, i.e., $R = 1.3 \times 10^{-11}$. The maximum response quantities and the maximum required control force are listed in columns (6) and (7) of Table 2, denoted by "Case II". A comparison of columns (4) and (5) with columns (6) and (7) indicates that it is not necessary to install displacement and velocity sensors on the upper structure. This conclusion is very important because it is expensive to install sensors. The same conclusion is also obtained in Ref. [14] by Yang through a numerical simulation for an eight-story seismic-excited building structure with rubber-bearing isolation system.

Here consider the case with existence of time delay in the control system. Assuming the delayed time is 0.3 s. As such, in the above, only the base floor is installed with displacement and velocity sensors. The **Q** matrix is assigned to be the same as that in the above, i.e., $Q(7,1) = 0.9$ and $Q(7,7) = 3.4$ with other elements being zero. The scalar R is chosen to be $R = 0.48 \times 10^{-9}$. Using the proposed control method, the maximum response quantities and the maximum required control force are shown in columns (8) and (9) of Table 2, denoted by "Case III". It is observed from the comparison of columns (8) and (9) with columns (6) and (7) that the control performance of the proposed method is close to that without the existence of time delay except that the maximum required control force is bigger. The control method proposed is effective in reducing the maximum responses of the structure. Then the scalar R is adjusted only such that the maximum required control force is the same as the case of columns (6) and (7), i.e., $\mathbf{U}_{max} = 35.99$ kN; the numerical results are shown in columns (10) and (11) of Table 2, denoted by "Case IV". A comparison of columns (10) and (11) with columns (6) and (7) indicates that, under the same control cost, the maximum response quantities of the structure when using the proposed control method are a little bigger than that without existence of time delay. But the values shown in columns (10) and (11) are much smaller than that in columns (2) and (3).

Fig. 4 shows the time histories of the base floor unit under the following three control strategies: without control for the structure, using the control method without existence of time delay (Case II of Table 2), and using the control method with the existence of the delayed time 0.3 s (Case IV of Table 2). Fig. 5 shows the time histories of the sixth story unit under the above three control strategies. We can observe from Figs. 4 and 5 that the proposed control method is an effective control strategy for dealing with the time delay in a vibration control system.

Our extensive simulation also indicates that both the system instabilities occur when the controller designed in the case of no time delay is used for the structure with the delayed time 0.3 s in control not only for the case of every degree of freedom of the structure being installed with sensors but also for the case of only the base floor being installed with sensors. Again, the control system with the time delay becomes unstable when the time-delay compensation method is used.

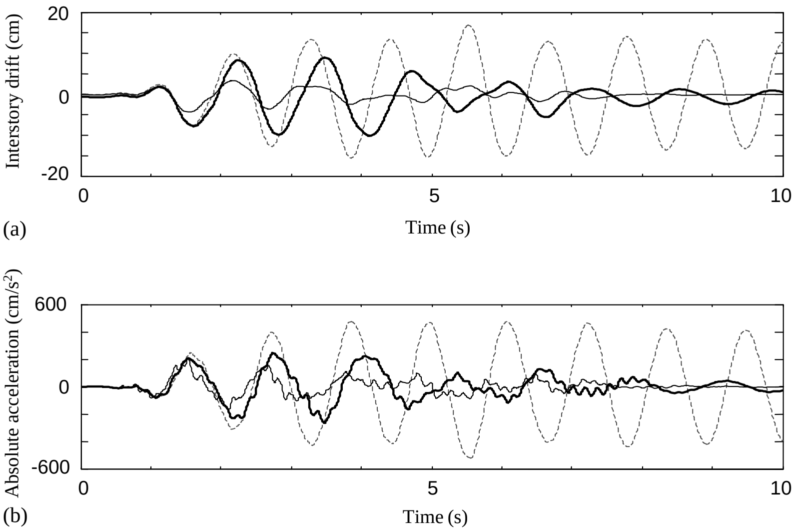

Fig. 4. Time histories of the base floor unit under the following three control strategies: no control, instantaneous optimal control without time delay and instantaneous optimal control with time delay: (a) interstory drift: no control ----------; optimal control without time delay ——; optimal control with time delay ——; (b) absolute acceleration: no control ---------; optimal control without time delay ——; optimal control with time delay ——.

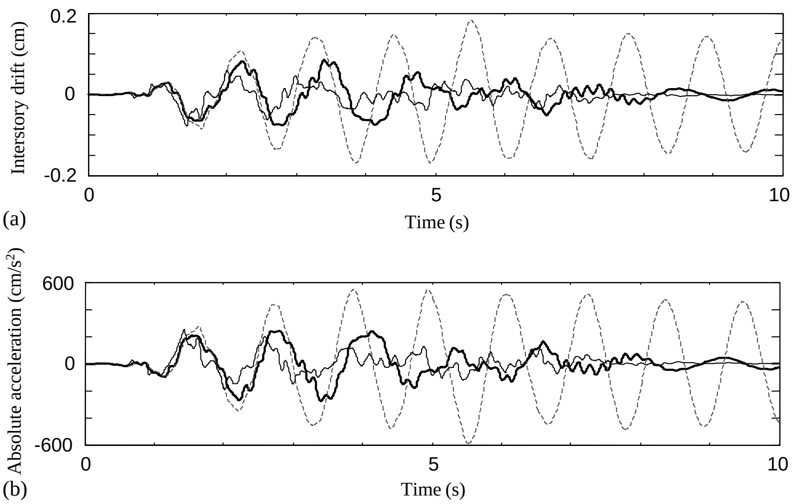

Fig. 5. Time histories of the sixth story unit under the following three control strategies: no control, instantaneous optimal control without time delay and instantaneous optimal control with time delay: (a) interstory drift: no control ---------; optimal control without time delay ——; optimal control with time delay ——; (b) absolute acceleration: no control ---------; optimal control without time delay ——; optimal control with time delay ——.

5. Conclusion remark

In an actively controlled system, time delay exists inevitably. Neglecting the time delay may result in instability of the control system. Therefore, it is better to take the time delay into account in control design before the active control is practically implemented. In this paper, based on the computation algorithm of the regular fourth order Runge–Kutta method, the instantaneous optimal control method with time delay in control is investigated. Since the obtained optimal controller contains integral term, which is not convenient for control implementation, the numerical algorithm for this integral term is investigated too. Because the optimal controller is obtained directly from the time-delay differential equation, system stability can be guaranteed easily. Thus this control method can be applicable to the case of large time delay. Numerical simulation results indicate that the control method proposed is effective in reducing maximum responses of vibration system. The vibration system may suffer from system instability if the time delay is neglected in control design.

Finally, it is worthy of mention that feedback gain in time-delay control system may possibly cause the happening of self-excited vibration [23]. Therefore, the choices of the weighting matrices **Q** and **R** given in Eq. (10) should be tried carefully in control design to avoid the happening of self-excited vibration.

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